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ABSTRACT

The effects of negative instances in the acquisition of the conjunctive concepts of distributivity and homomorphism were examined. Ninety-two elementary education majors were used as subjects. Two treatment levels for distributivity (series of positive instances or positive and negative instances) and the same treatment levels for homomorphism were crossed to form a 2 x 2 factorial design with 23 subjects per cell. Criterion variables were number of correct classifications, stimulus intervals, and postfeedback intervals. All pretests, treatments, and posttests were administered using computer terminals. The results supported the hypotheses that negative instances enhance conjunctive concept acquisition and that effects of negative instances for one concept transfer to another concept. (Author/DT)

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POSITIVE VERSUS POSITIVE AND NEGATIVE INSTANCES AND THE ACQUISITION OF THE CONJUNCTIVE CONCEPTS OF DISTRIBUTIVITY AND HOMOMORPHISM[1]

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Negative instances have been considered by mathematicians to be essential to the understanding of advanced mathematical concepts (Gelbaum and Olmsted, 1964; Steen and Seebach, 1970). Dienes (1964) argues for the use of negative instances in the teaching of mathematics to elementary and secondary school children. Educational psychologists have stated explicitly that all instructional sequences designed for concept learning should include negative instances (Bereiter and Engelman, 1966; Markle and Tiemann, 1960). A review of the research in experimental psychology generally supported a deleterious effect for negative instances in conjunctive concept learning, but for nonconjunctive concepts the use of negative instances was sometimes advantageous (Clark, 1972; Bourne and Dominowski, 1972).

Two questions were examined: 1) What are the different effects of an instructional sequence of positive and negative instances and a sequence of all positive instances in the acquisition of the conjunctive concepts of distributivity and/or homomorphism; and 2) Assuming there are effects for negative instances, do the effects of negative instances for one concept transfer to another concept?

Research in mathematical concept acquisition generally supports the use of negative instances to improve concept acquisition. In a

classroom study, Shumway (1971) found that negative instances discouraged overgeneralization errors by 13 and 14 year old Ss for concepts involving the properties of binary operations. Using programmed instruction, Marine (1972) found results favoring negative instances and Dossey (1972) found deleterious effects for negative instances. Shumway (1972 a) using computer terminals to present treatments for comutativity and associativity to 14 and 15 year old Ss, not only found results favoring treatments containing negative instances, but also found that the effect for negative instances transferred from one concept to another. Several alternate explanations for the results were proposed. There were near significant results for stimulus intervals and postfeedback intervals ($p < .07$). Differences in time variables could account for the advantage cited for negative instances. The Ss were not remarkably successful on the criterion measure. Differences could be attributed to the Ss maintaining the same proportion of positive and negative instances during criterion measure as was present during the treatment and simply guessing. The results may be unique to the conjunctive concepts of commutativity and associativity. It was proposed that a similar study be conducted with different concepts and Ss of an age older than 14 and 15 to further investigate these questions.

Method[2]

Ninety-two elementary education majors enrolled in a required mathematics course at Ohio State University were randomly assigned in equal numbers to four treatments. The course was the second of two mathematics courses designed to explore the mathematical concepts

taught in elementary school. Most subjects were college sophomores or juniors.

Concept A was defined to be distributivity of a binary operation over a binary operation and Concept B was defined to be homomorphism of a function over a binary operation. The symbol A+ denoted a treatment of 10 positive instances of Concept A and the symbol A$\pm$ denoted a treatment of five positive instances and five negative instances of Concept A. The symbols B+ and B$\pm$ were defined similarly. Figure 1 specifies the 2 x 2 design matrix. Each treatment consisted of 20 instances in a fixed but random order.

Insert Figure 1 here

Table 1 specifies the number and type of instances for each treatment.

Insert Table 1 about here

A sample instance during the treatment was as follows:

Stimulus:

1. a @ b = 3 * a * b   2 @ 5 = 30,   4 @ 1 = 12.

   a o b = a + b   4 o 7 = 11,   6 o 2 = 8.

   a @ (b o c) = (a @ b) o (a @ c) ?

Response:

y

Feedback:

Correct.

Response:

Hit 'return key' to receive next stimulus.

The stimulus interval was taken to be the length of time between the end of the typing of the stimulus, i.e., the symbol "?," and the entering of the symbol "y," the response. There was no delay of the informative feedback. As soon as the response was entered, the feedback was typed. The postfeedback interval was taken to be the length of time between the typing of the feedback and the subject's hitting of the return key to receive the next stimulus. Both concepts were infinite conjunctive concepts defined over infinite classes (Shumway, 1972). The paradigm was the reception paradigm; the task was classified as rule or principle learning rather than attribute learning, as the attribute was identified for the Ss. The major criterion variable was the number of correct classifications of new instances presented after the treatment.

The treatments were administered with an IBM 370/155 computer and IBM 2741 computer terminals. The programming language was Coursewriter III, Version 2 (IBM, 1969). Stimulus intervals and postfeedback intervals for each item were recorded as well as the student's responses.

For the calculational pretests PCA and PCB, Ss were asked to carry out a numerical calculation for each of the binary operations and functions to be used in the posttests (POA and POB). Stimulus intervals were recorded during both pretests. The items for PCA and PCB were randomly ordered.

The posttests for Concept A and Concept B, POA and POB, each consisted of five positive instances and five negative instances not given in any of the treatments. Ss were asked to classify each instance as during the treatments, but no informative feedback was given.

Figure 2 gives a flow chart of the complete experimental sequence.

There were three sessions at the computer terminal. The first session, lasting approximately 15 minutes, consisted of an introduction to binary operations and functions and the pretests PCA and PCB. The second session, lasting approximately 20 minutes, consisted of a brief introduction and one of the four experimental treatments. The third session, lasting approximately 15 minutes, consisted of the two posttests, POA and POB. Approximately two-thirds of the Ss completed all three sessions in one sitting. In all cases Ss completed all three sessions within seven days. Computer terminals were available at many locations on campus and Ss could use any available terminal between 8 a.m. and 11 p.m.

Insert Figure 1 about here

The independent variables were:

1. Levels of A (A+ or A±);
2. Levels of B (B+ or B±);
3. PCA - Pretest, calculations with operations;
4. PCB - Pretest, calculations with functions;
5. PCSIA - Total Stimulus Interval for PCA;
6. PCSIB - Total Stimulus Interval for PCB.

The dependent variables were:

1. POA - Posttest for Concept A;
2. POB - Posttest for Concept B;
3. POSIA - Total Stimulus Interval for POA;
4. POSIB - Total Stimulus Interval for POB;
5. TSIA - Total Stimulus Interval during Treatment for Concept A;
6. TSIB - Total Stimulus Interval during Treatment for Concept B;

7. TPIA - Total Postfeedback Interval during Treatment for Concept A;
8. TPIB - Total Postfeedback Interval during Treatment for Concept B.

Results

The data were analyzed using the Clyde MANOVA program (Clyde, 1969) for a multivariate two-way analysis of covariance. Because of the symmetry of the design, the results for Concept B were viewed as a potential replication for the results for Concept A. Hence, the analysis for Concept B was done separately from the analysis for Concept A. Achievement variables were separated from time variables.

The pretests PCA, PSIA, PCB, and PSIB were subjected to multivariate and univariate analysis of variance. No significant differences were found (in all cases $\underline{p} > .1$). Mean scores on PCA and PCB were in excess of 89%. $\underline{S}$s were able to calculate with the operations and functions which were to appear on the posttests.

While no pretest differences were significant, covariance procedures were chosen for the analysis to increase the power of the tests and because there was a clear conceptual relationship between pretests and posttests.

The variable POA, as the major criterion variable for Concept A, was analyzed using a univariate analysis of covariance with PCA as covariate. Table 2 summarizes the results of the analysis of POA. There was a significant effect for levels of A (A+, A$\pm$) favoring A$\pm$ ($\underline{p} < .05$).

Insert Table 2 here

Figure 3 displays the adjusted cell and margin means and a plot of the cell means.

Insert Figure 3 here

The variable POB, as the major criterion variable for Concept B, was analyzed using a univariate analysis of covariance with PCB as covariate. Table 3 summarizes the results of the analysis of POB. There was a significant interaction between levels of A and levels of B ($p < .05$).

Insert Table 3 here

Figure 4 displays the adjusted cell and margin means and a plot of the cell means. The interaction was tested for disordinality. While a *t* test showed that the cell mean for A+B+ was significantly lower than the cell mean for A±B+ ($t = 2.03$, $df = 44$, $p < .05$), there was no evidence that the cell mean for A+B± was significantly higher than the cell mean for A±B± ($t = 1.24$, $df = 44$, $p > .2$). Hence, the interaction effect was not classified as a disordinal interaction. It appears that negative instances for Concept A improved performance on Concept B when no negative instances for Concept B were present. Transfer occurred.

Insert Figure 4 here

The posttest time variables for Concept A (TSIA, TPIA, POSIA) and Concept B (TSIB, TPIB, POSIB) were subjected to multivariate and univariate analysis of covariance using PCSIA or PCSIB as covariate. None of the multivariate or univariate tests were significant ($p < .05$).

Conclusions

Two questions were examined.

1. What are the different effects of an instructional sequence of all positive instances and a sequence of positive and negative instances on the acquisition of the conjunctive concepts of distributivity and/or homomorphism?
2. Do effects for negative instances on the acquisition of one concept transfer to the acquisition of another concept?

Question 1 was answered as follows: For the acquisition of the concept of distributivity a sequence of positive and negative instances was favored over a sequence of all positive instances.

Question 2 was answered as follows: There was an interaction effect between negative instances for distributivity and negative instances for homomorphism. The effect of negative instances for distributivity improved performance on homomorphism when no negative instances for homomorphism were present. Transfer occurred.

Discussion

In a study of similar design using the concepts of commutativity and associativity, Shumway reported that negative instances improved performance and that the effects of negative instances transferred from one concept to another (Shumway, 1972a). However, it was not clear that there was not a difference in the time variables of stimulus interval and postfeedback interval which could have accounted for the differences found. Subjects performed at a level no better than that expected by chance alone and did not show marked ability with the pretest calculations.

Several of the limitations of the study by Shumway (1972a) were not found in this study. There were no significant multivariate or univariate time differences. Subjects were not guessing. Subjects' performance was at a level better than expected by change alone and Ss exhibited a great deal of ability with the pretest calculations. Nevertheless, the results again supported the conclusion that a treatment of negative and positive instances improved concept acquisition and that the effect of negative instances transferred from one concept to another.

The results support the research strategy taken by this author. Negative instances have been shown to be an important variable in laboratory concept acquisition. For unidimensional and conjunctive concepts, negative instances are generally deleterious. For disjunctive, conditional, and biconditional concepts, negative instances enhance concept acquisition (Clark, 1971). In order to generalize such results to concepts recognized for their social value, for example, concepts in the school curriculum, it is necessary to perform studies which attempt to replicate the laboratory results with concepts from the school curriculum. It appears that although distributivity and homomorphism can be classified as conjunctive or possibly even unidimensional concepts (Shumway, 1972b), the increase in the size of the class over which the concept is defined to infinity and the modification of the instructional sequence to actually giving the subject the attribute to test sufficiently complicates the task so that, contrary to laboratory evidence, negative instances improve subjects' performance. It seems appropriate to begin to study, in detail, relationships between variables identified

in the classical concept formation studies and concepts such as distributivity and homomorphism.

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FOOTNOTES

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2. Further details concerning methods, results, and related literature may be found in Shumway, 1972b.

TABLE 1

Number and Type of Treatment Instances per Cell and Number of Subjects per Cell

Cell	Treatment	Number and Type of Instances				Total	Number of Subjects Per Cell
		Concept A		Concept B			
		Positive	Negative	Positive	Negative		
11	A+B+	10	0	10	0	20	23
12	A±B+	5	5	10	0	20	23
21	A+B±	10[a]	0	5	5	20	23
22	A±B±	5[a]	5	5	5	20	23

[a]Item 12 for cell 21, and item 3 for cell 22 were scored as positive instances and the subjects received feedback which identified the instance as positive. In fact, however, the instance was actually negative. An examination of the response patterns revealed no discernible disruptive influence. It was assumed that the programming error would reduce the chances for different effects for treatments. The item was treated as a positive instance in the analysis.

TABLE 2

Analysis of Covariance for Concept A of POA using PCA as Covariate

Source	df	F	p
Equality of Regression	3,84	1.246	.298
Regression	1,87	5.407	.022*
A x B	1,87	0.967	.328
A	1,87	4.216	.043*
B	1,87	1.070	.304

POA - Posttest for Concept A.
PCA - Pretest, Calculations with operations.
*$p < .05$

TABLE III

Analysis of Covariance for Concept B of POB using PCB as Covariate

Source	df	F	p
Equality of Regression	3,84	2.364	.077
Regression	1,87	10.501	.002**
A x B	1,87	4.984	.028*
A	1,87	1.208	.275
B	1,87	.138	.711

POB - Posttest for Concept B
PCB - Pretest, Calculations with functions.
**$p < .01$, *$p < .05$

FIGURE CAPTIONS

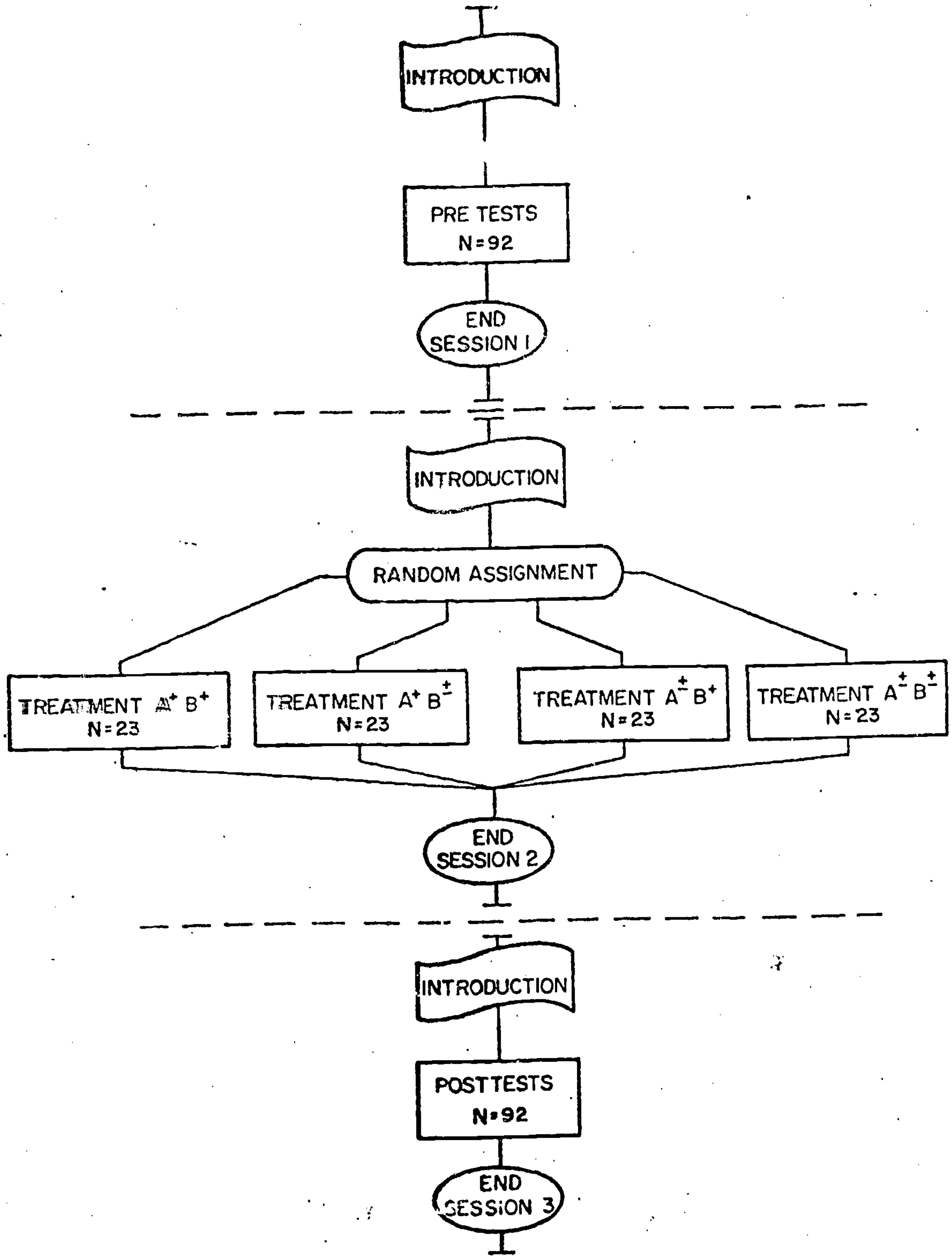
Figure 1. Design matrix

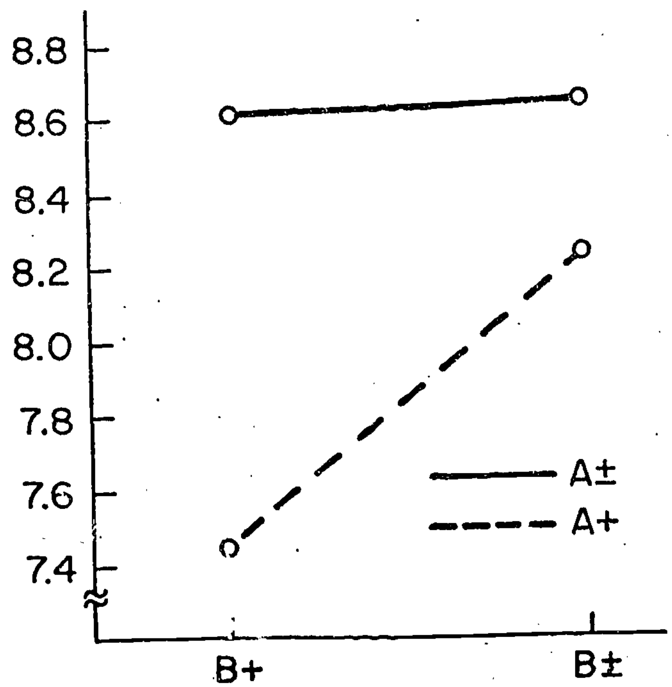
Figure 2. Flow chart of Experiment

Figure 3. Adjusted means for POA (A effect)

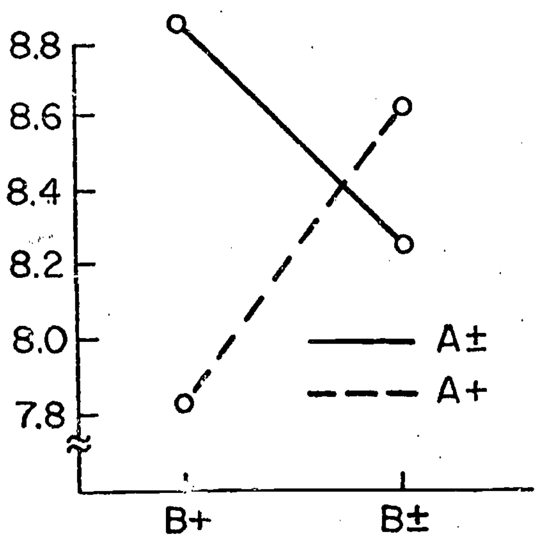
Figure 4. Adjusted means for POB (AxB effect)

	A+	A±
B+	A+B+	A±B+
B±	A+B±	A±B±


INTRODUCTION
PRE TESTS
N=92
END
SESSION I
INTRODUCTION
RANDOM ASSIGNMENT
TREATMENT $A^+ B^+$
N=23
TREATMENT $A^+ B^\pm$
N=23
TREATMENT $A^\pm B^+$
N=23
TREATMENT $A^\pm B^\pm$
N=23
END
SESSION 2
INTRODUCTION
POSTTESTS
N=92
END
SESSION 3



	A+	A±	
B+	7.475	8.601	8.038
B±	8.218	8.620	8.419
	7.846	8.610	



	A+	A±	
B+	7.797	8.847	8.322
B±	8.615	8.262	8.439
	8.206	8.555	